Lesson 8:
Out To Launch

OBJECTIVE:

Students calculate “area under the curve” to determine the distance a spacecraft has traveled in the first two minutes after liftoff.

NATIONAL STANDARDS

Mathematics

Algebra
- analyze change in various contexts

Geometry
- specify locations and describe spatial relationships using coordinate geometry and other representational systems

Measurement
- understand measurable attributes of objects and the units, systems, and processes of measurement
- apply appropriate techniques, tools, and formulas to determine measurements

Problem Solving
- build new mathematical knowledge through problem solving
- solve problems that arise in mathematics and in other contexts
- monitor and reflect on the process of mathematical problem solving

Communication
- organize and consolidate mathematical thinking through communication
- communicate mathematical thinking coherently and clearly to peers, teachers, and others
- use the language of mathematics to express mathematical ideas precisely

Connections
- recognize and use connections among mathematical ideas
- understand how mathematical ideas interconnect and build on one another to produce a coherent whole
- recognize and apply mathematics in contexts outside of mathematics

Representation
- create and use representations to organize, record, and communicate mathematical ideas
- select, apply, and translate among mathematical representations to solve problems
- use representations to model and interpret physical, social, and mathematical phenomena

Science

Unifying concepts and processes in science
- Evidence, models, and explanation

Science as inquiry
- Understanding about scientific inquiry

Science and technology
- Abilities of technological design
- Understanding about science and technology

History and nature of science
- Science as a human endeavor
- Nature of scientific knowledge
- Historical perspectives

Technology

Standard 17 - Students will develop an understanding of and be able to select and use information and communication technologies.

MATERIALS:
- Computer with Internet connection connected to a projector
- Graphing calculators or computer-based modeling software (for students)
- One hand out, sheet of graph paper, and ruler for each student

Last Launch of Space Shuttle - Atlantis (STS-135) - July 8, 2011

BACKGROUND INFORMATION:

Consider allowing precalculus students an opportunity to experience the underpinnings of integral calculus. This activity was created with that goal in mind. Students will experience computing the "area under a curve" and attempt to bring meaning to the outcome.

Consider presenting the activity to students as a semester project wherein they may attempt modeling of actual data and try to make sense of it. The activity is appropriate for students at virtually all high school levels as long as they are comfortable with graphing & modeling data, familiar with power functions, and have some background (or facility) with dimensional analysis.

Area formulae

- Left endpoint approximation: $\sum_{i=1}^{n} f(x_{i-1})\Delta x$
- Right endpoint approximation: $\sum_{i=1}^{n} f(x_i)\Delta x$
- Midpoint approximation: $\frac{b-a}{n}\sum_{i=1}^{n} f(\frac{x_{i-1}+x_i}{2})$
  where *a* and *b* are the endpoints of the interval
- Trapezoidal approximation: $\frac{b-a}{2n}\sum_{i=1}^{n} f(x_i)$
  where *a* and *b* are the endpoints of the interval

PROCEDURE:

1. Discuss why the label "rocket scientist" has long been synonymous with "stellar" problem solving capabilities. If "it's not rocket science" means "you can do this" then "it's rocket science" probably means "only the best thinkers can do this."
2. Remind students the basic form of a power model and how one differs from the others (the general form for power functions is $y = ax^b$). Consider reminding students how to derive the coefficient if the power is given (or known).
3. Lead a discussion on how students would "compute the areas under a curve" if the equation were $y = 2x + 3$; $0 = x = 4$. Either demonstrate this or have students perform the area calculation independently. Hopefully, they will quickly determine that the shape is merely a trapezoid, and the area can be calculated easily.
4. Extend the discussion to how students would "compute the areas under a curve" if the equation were $y = 2x^2 + 3$; $0 = x = 4$. Place emphasis on how the "curve" makes the job a bit more complicated than for a line. Lead students to understanding they can approximate the area with small trapezoids and add them up.
5. Show the Space Shuttle launch (**http://www.youtube.com/watch?v=4FROxZ5i67k**) and explain how the Solid Rocket Boosters (SRBs) detach about two minutes after launch. From there, the main engines lift the Orbiter to its orbital altitude. Tell the class tol assume the Shuttle ascends pretty much vertically (which is, of course, not true).

Solid Rocket Boosters detach from Shuttle

6. Distribute the *Out To Launch* worksheet, graph paper, and rulers.
7. Monitor as students draw the interval lines and begin computing areas. The result should be around 173,000 feet.
8. Assure all students have completed all area computations, have a total, and have addressed the questions.
9. Discuss the answers on the paper and explore the variability of students' area approximations. Ask if the approximation would be an underestimate or over estimate of the Shuttle altitude at SRB separation.
10. Conclude by announcing to students that they are now on their way to becoming rocket scientists!

Note: A reasonable squared power function is $y = (4151 \div 125^2)x^2$ where x is the number of seconds after launch and y is the speed of the Space Shuttle in feet/second. This is reasonable because a rocket's acceleration should increase in an increasing rate because of lost mass during combustion, the curve should go through (0, 0) and (125, 4151), and the "approximation" the model provides will miss the values where the shuttle engines were throttled back; hence, the model provides the speed at 125 seconds if the engines delivered constant thrust. The area, therefore, is the altitude at SRB separation (assuming the Shuttle ascends vertically).

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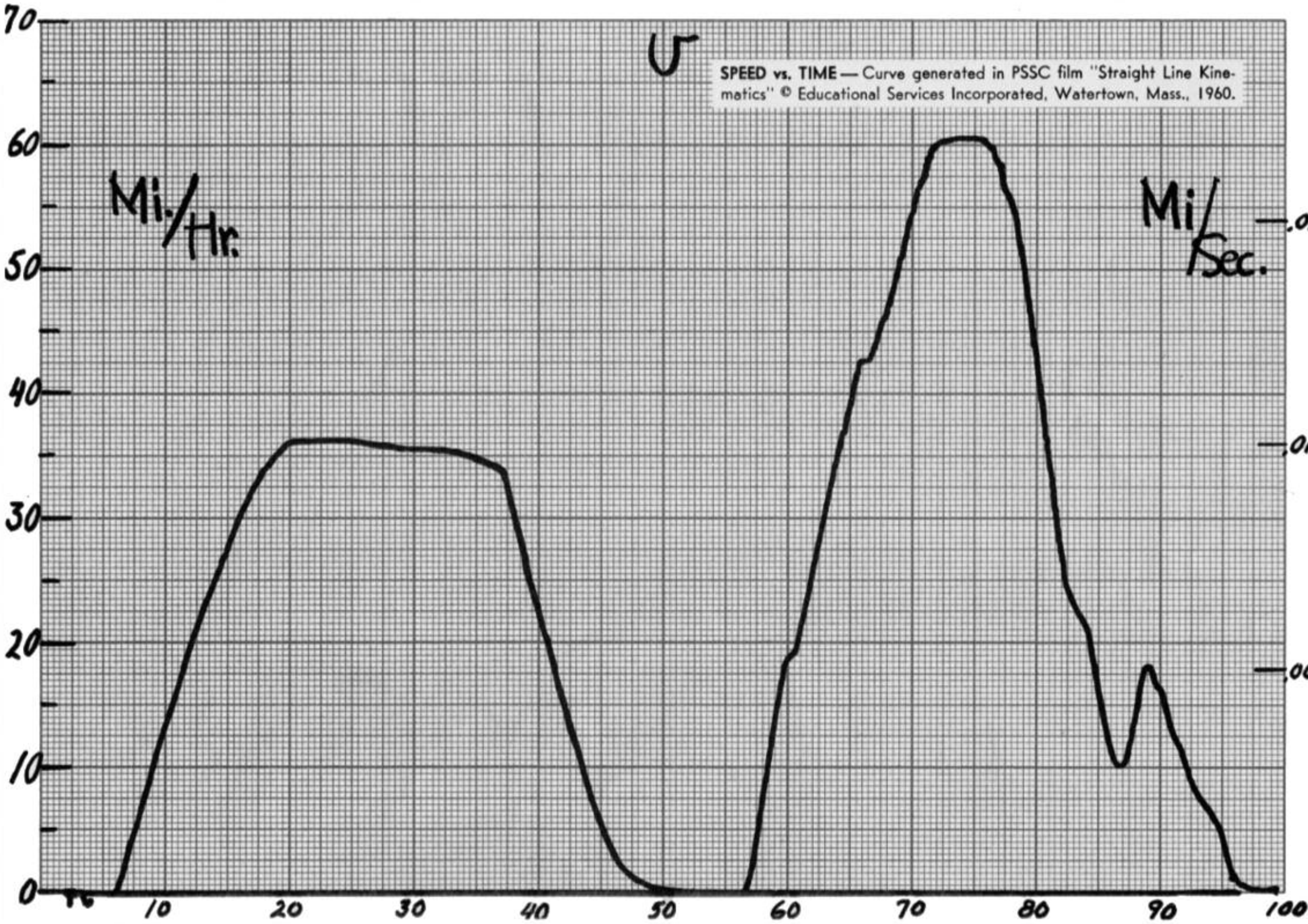
Challenge Problem

Velocity vs. time for a 1961 Thunderbird in a test area



Determine the distance the car has driven.
Watch out for the units (miles per hour are on the vertical axis and number of seconds on the horizontal axis).

SUMMARY:

Integral calculus need not be mysterious nor laden with cryptic formulae; it can be understood from the perspective of performing arithmetic (appropriately) on the graphical representation of an outcome. If the units on the axes support the computation (e.g., meters/second * number of seconds = number of meters) adding the areas of the representative columns (the "Riemann Sum") gives an approximation of the true area under the curve. Using narrower columns gives an even better estimate and making them infinitesimal in width is calculus!

EVALUATION:

See the Challenge Problem. (Page 49)

LESSON ENRICHMENT/EXTENSION:

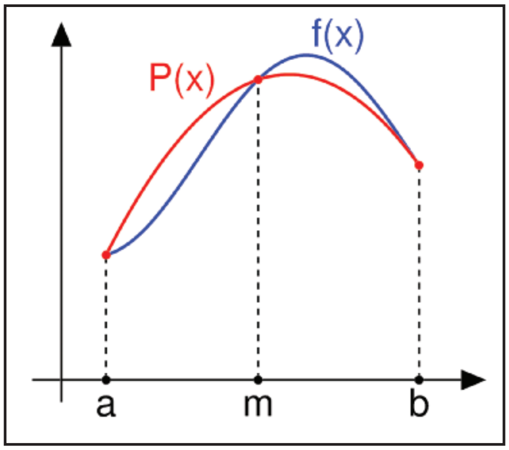
- Secretly produce four versions of the activity, each using a different method for calculating the areas of the intervals—rectangles using left endpoint, right endpoint, or midpoint for the height of the rectangle; or, trapezoids using both endpoints. Consider producing them on different colors of paper and having students with the same color of paper work together. Have students compare answers during the activity debrief, see if they can explain why the answers differ, and, for this function, which method *should* provide the closest approximation.
- Have different groups approximate the area using intervals of different widths. Then, compare the approximations. Draw students to the realization that the approximate approaches the correct value for the area the more intervals are used.
- Use a spreadsheet to calculate the areas and explore changing interval widths.
- Connect to Riemann Sum for beginning calculus students. (A Riemann Sum is a method for approximating the total area underneath a curve on a graph, otherwise know as an integral. It may also be used to define the integration operation. The method was named after German mathematician, Bernhard Riemann.)
- Connect to Simpson's Rule for advanced calculus students. (Simpson's rule is a staple of scientific data analysis and engineering. It is widely used, for example, by Naval architects to calculate the capacity of a ship or lifeboat.)

Space Shuttle Launch

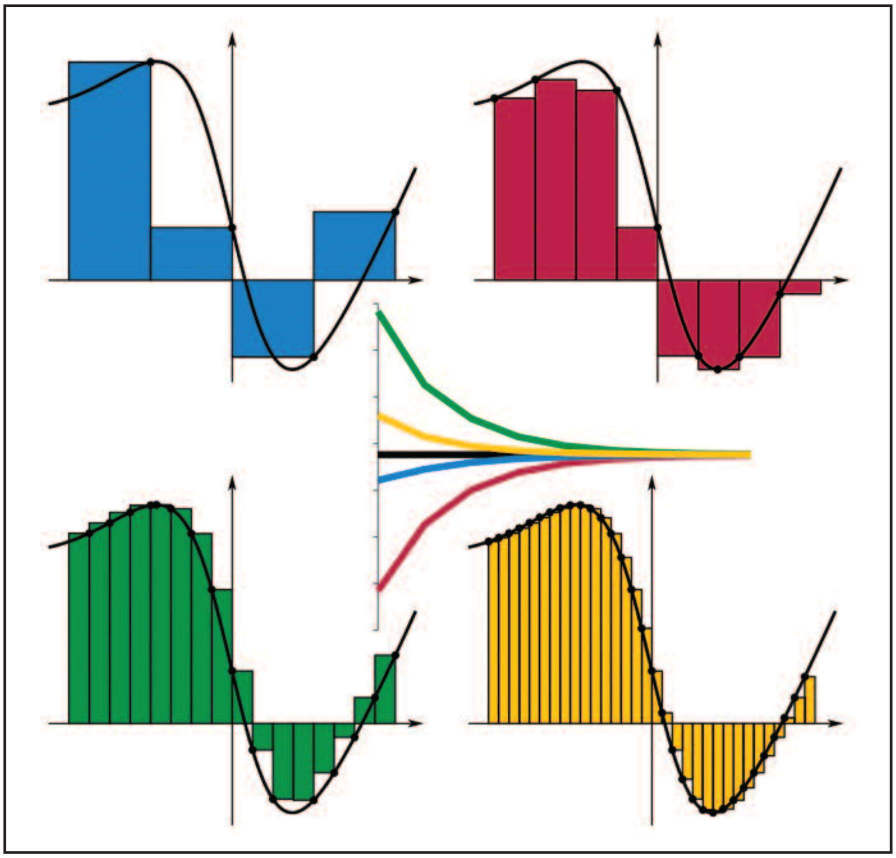
ASSOCIATED WEBSITES AND/OR LITERATURE:

- You Tube video of shuttle launch: **http://www.youtube.com/watch?v=4FROxZ5i67k**

- Riemann Sum calculator: **http://mathworld.wolfram.com/RiemannSum.html**



Simpson's Rule



Riemann Sum Convergence

Out To Launch

Name: ____

On 7 May 1992, the space shuttle Endeavour was launched on mission STS-49, the purpose of which was to install a new perigee kick motor in an *Intelsat* communications satellite. The following table gives the velocity data for the shuttle between liftoff and the jettisoning of the solid rocket boosters (SRBs).

Event	Time(s)	Velocity (ft/s)
Launch	0	0
Begin roll maneuver	10	185
End roll maneuver	15	319
Throttle to 89%	20	447
Throttle to 67%	32	742
Throttle to 104%	59	1325
Maximum dynamic pressure	62	1445
SRB separation	125	4151

Launch of STS-49, Endeavour, on 7 May 1992

1. Make a scatter plot of the data using a full sheet of graph paper.

2. Model the data as a squared power function and draw the function on the same graph as the scatter plot.
   Note: this is NOT the same as a quadratic function!

3. Explain why this particular model is best/most appropriate for the context of a shuttle launch.

4. Draw vertical lines on the graph every 10.0 seconds and note where they intersect the graph of the model.
   Note that the last interval will be half the normal width, so the last vertical line will be at x = 125.

5. Determine the heights of the sides of each interval and transcribe the measurements into the "data table"

6. Substitute the values from #5 into an appropriate formula and calculate the area of each region obtained in #4.

7. Add up all the interval areas. Report the value with appropriate units.

8. Answer the question, "What is the significance of the area under the curve of the shuttle launch data?"

9. What advantage would using intervals 5.0 seconds wide for the entire graph provide?

Interval	Left height	Right height	Width	Area Formula (with substitutions)	Area
10					
20					
30					
40					
50					
60					
70					
80					
90					
100					
110					
120					
125					

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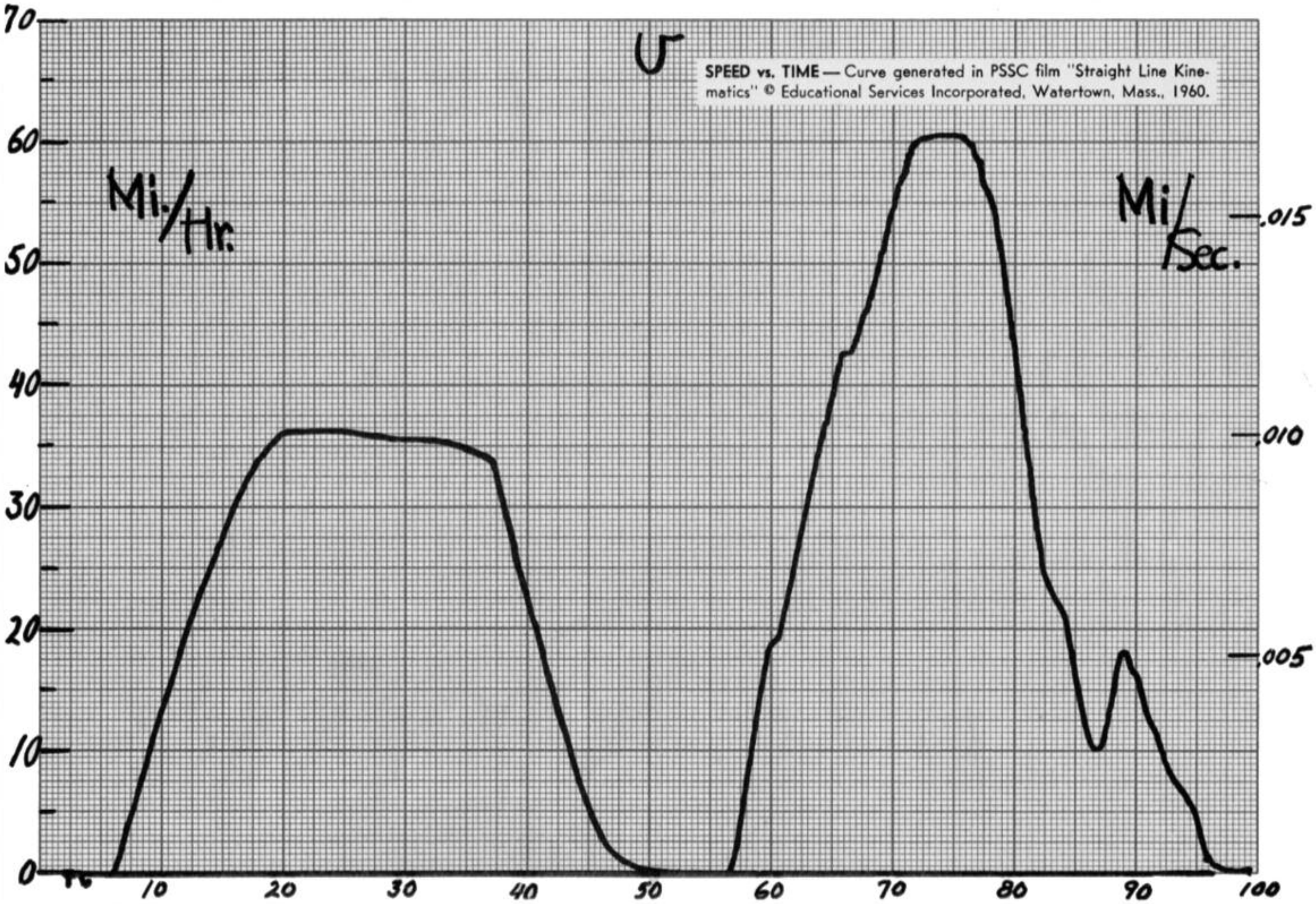

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